A median centred difference gradient operator and its application in watershed segmentation

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A new gradient operator for greyscale images is proposed whose output is based on the median centred difference of the intensities within a local mask. The operator is a simplified form of the Robust Colour Morphological Gradient and, as such, is robust to impulsive noise and provides well localised gradient estimates. A quantitative evaluation using simulated images shows that the new operator is particularly suitable for the watershed transform, producing significantly better watershed segmentation results than conventional edge operators such the morphological gradient and the Sobel.

Introduction

Estimating image gradient is one of the most fundamental low-level image processing operations. Recently, colour edge detection has received much attention. One such colour edge detector is the Robust Colour Morphological Gradient (RCMG), a low complexity, high performing gradient operator for multichannel images that is also very robust to image noise [1]. Part of the motivation for the RCMG was to enable the morphological gradient to be applied to multichannel images by overcoming the lack of the explicit ordering required by the erosion and dilation operations. The RCMG also introduced the concept of using the median centred difference (MCD) of the vectors within a local mask as a robust gradient estimate for colour images. This letter proposes the use of the MCD of the intensities within a local mask as an edge detector for greyscale images. Using order statistics, a new analysis of the MCD operator for greyscale images is presented, leading to a simple greyscale MCD definition that is suitable for an efficient implementation. As the operator is a single channel form of the RCMG it inherits the noise robustness demonstrated by the RCMG.

Although the new operator can also provide an estimate of edge direction, as acknowledged in [1], the estimate does not have a high accuracy. Here, the quality of its gradient magnitude is objectively evaluated by investigating its segmentation performance in conjunction with the watershed transform. In particular, images corrupted by channel noise present significant challenges for subsequent tasks such as edge detection and segmentation [2]. Channel noise is typically modelled as impulsive noise and this is the focus of our quantitative evaluation.

Median Centred Difference (MCD) gradient operator

The proposed edge operator is a single channel form of the RCMG. The RCMG is defined using vector differences, as this overcomes the absence of an explicit ordering for vector data. In its single channel form the pixel intensities can be unambiguously ordered, allowing the median centred difference to be defined using order statistics. Given N intensities from within a local window, let the i^{th} smallest value be denoted $x_{(i)}$ such that

$$x_{(1)} < x_{(2)} < \cdots < x_{(N)} . \tag{1}$$

To determine the MCD, consider the situation where a window contains a step edge of any orientation passing through the centre of the window. The intensities within the window can be viewed as coming from the two populations on either side of the edge. Of these, the majority population consists of all the pixels on the side of the edge that contains the central pixel and the remaining pixels will form the minority population. An odd-length window of size $N = (2k+1)$ x (2k+1) will therefore contain k x $(2k+1) = 2k^2+k$ pixels from the minority population. Each pixel in the minority population can form a centred difference across the edge with one of the pixels from the majority population such that there are $2k^2+k$ centred differences, given by

$$\left| x_{(N-r)} - x_{(r+1)} \right| \qquad r = 0, 1, \cdots, \left(2k^2 + k\right) - 1 \tag{2}$$

where r is the number of pairs of pixels to remove before finding the centred difference. When $r = 0$ the well known morphological gradient (MG) [3] results. The MCD edge detector can be defined by setting $r = \left\lfloor \left(2k^2 + k - 1\right)/2 \right\rfloor$ and provides a good estimate of the edge magnitude [1]. In the presence of noise the robustness of the edge operator can be improved by increasing value of r, which has the effect of rejecting additional noise-corrupted pixels. Providing $r < 2k^2+k$, at least one pixel from the minority population will be retained when calculating the centred difference using (2). For the colour image case with a 5 x 5 mask, setting $r = 8$ was demonstrated to produce good results over a wide range of noise levels [1].

The simple order statistics definition of the MCD given by (1) and (2) enables an efficient implementation of the MCD edge detector, based on the fast two-dimensional median filtering algorithm of Huang et al. [4]. The technique in [4] is based on storing the intensity histogram of the pixels within the window and updating it as the window moves, while keeping track of how the position of the median changes during the updating. This approach can easily be adapted to track the positions of the intensities ranked $x_{(r+1)}$ and $x_{(N-r)}$ as the window moves from one pixel position to the next, allowing the MCD output of (2) to be efficiently found. This efficient implementation is not possible if the RCMG definition from [1] is directly applied to greyscale images.

Image segmentation using the watershed transform

The watershed transform is a widely used technique for image segmentation [5 - 7]. In the watershed transform, an image is considered to be a topographic surface whose minima are flooded from below while maintaining the boundaries between different catchment basins. In practice, the watershed is applied to the image gradient and the watershed lines separate homogeneous regions, giving the desired segmentation result. The gradient image for the transform is often found using the MG [3]. However, noise in the gradient image results in over-segmentation which can have a significant adverse affect on the quality of the segmentation results. Although markers can be used to ameliorate the over-segmentation [7], the quality of the gradient estimate has a major influence on the segmentation performance.

While the MG is widely used, there has been little work evaluating the watershed performance with different gradient operators. The proposed MCD gradient operator inherits the good edge localisation and noise robustness of the RCMG. These properties, together with the fact that the watershed does not require gradient direction, make the MCD operator eminently suitable for use with the watershed transform. A quantitative evaluation of this approach is presented below.

Performance evaluation

To evaluate the performance of the MCD edge detector, it was applied to 16 simulated images (see Fig. 1 for an example image) and resulting gradient images used as inputs to the watershed transform. The simulated images were then corrupted by various levels of impulsive noise, so that the Peak Signal to Noise Ratio (PSNR) varied between 7 – 35 dB, and the watershed results for each noise-corrupted image compared to the noise-free segmentation results using Pratt's Figure of Merit (FOM) [8]. 8-connectivity was used in the watershed as this has been shown to be more robust to noise [9].

Fig. 2 presents the average FOM results for the 16 simulated images where, for each image, 1000 repetitions were averaged for each noise level. The MCD was applied using a 5 x 5 mask with r = 8 and, for comparison, the watershed results for the MG and the Sobel magnitude using both 3 x 3 and 5 x 5 masks are also shown. The Sobel operator produces the worst watershed segmentations over the 10 – 35 dB range, clearly demonstrating its sensitivity to impulsive noise. The MG performs better than the Sobel, particularly in the 10 – 35 dB range with a 3 x 3 mask and for 15 – 35 dB with a 5 x 5 mask. The MCD gradient produces the best FOM performance over the entire 7 – 35 dB range and significantly out-performs all other gradient operators between 7 - 20 dB. The advantage of the MCD increases with decreasing PSNR, for example at 10 dB its average FOM is 0.7 above that of the next best-performing gradient operator.

Fig. 3 illustrates the differences between the watershed segmentations of the 3 x 3 MG (Fig. 3a) and the MCD (Fig. 3b) at a PSNR of 20.18 dB. Although the FOM difference between the MG and the MCD is only 0.048, the latter is clearly preferable as its watershed lines show a close correspondence with the true edges and are only slightly corrupted by noise. In contrast, the MG watershed lines are significantly affected by the image noise and also exhibits spurious responses.

Conclusions

A MCD edge detector for greyscale images has been proposed that can be simply defined using order statistics. The operator is a single channel form of the RCMG and is robust to channel (impulse) noise. A quantitative evaluation shows that applying the watershed transform to MCD gradient images produces results that are significantly less sensitive to impulsive noise than those of either the Sobel or the MG. As such, the new operator is very suitable for use with the watershed transform.

References

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Figure captions:

Fig. 1. Example 256 x 256 simulated test image.

Fig. 2. Average Figure of Merit (FOM) [7] versus Peak Signal to Noise Ratio (PSNR) for 16 test images, with 1000 repetitions per image at each noise level.

Fig. 3. Example watershed transform results at a PSNR of 20.18 dB produced by (a) 3 x 3 Morphological Gradient (FOM = 0.898) and (b) Median Centred Difference (FOM = 0.946).

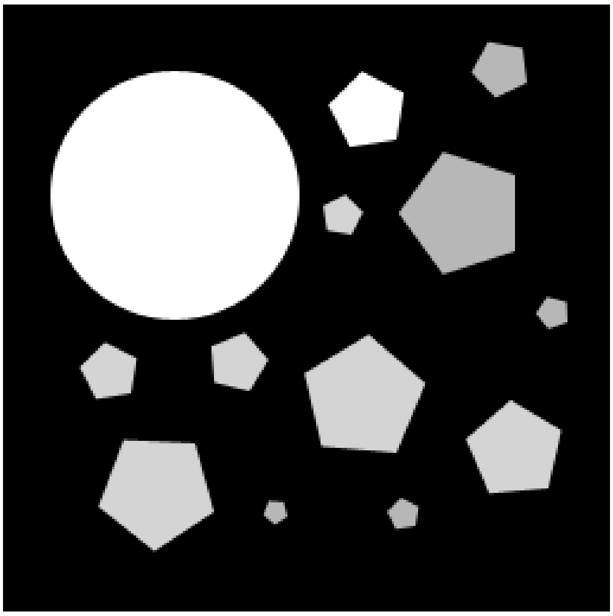


Fig. 1

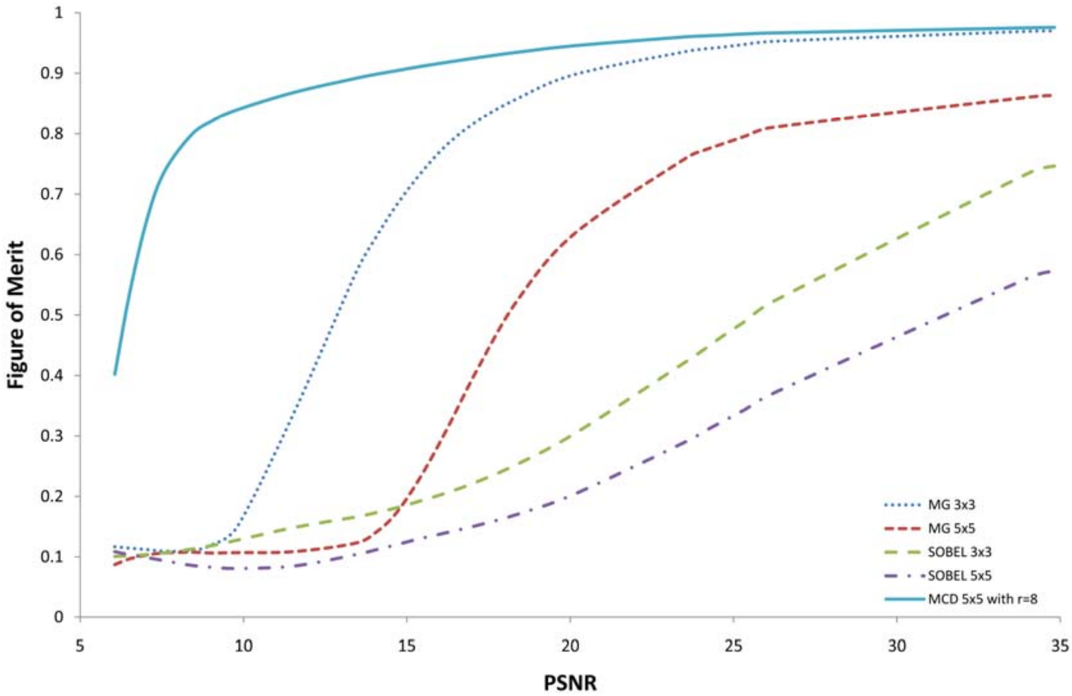
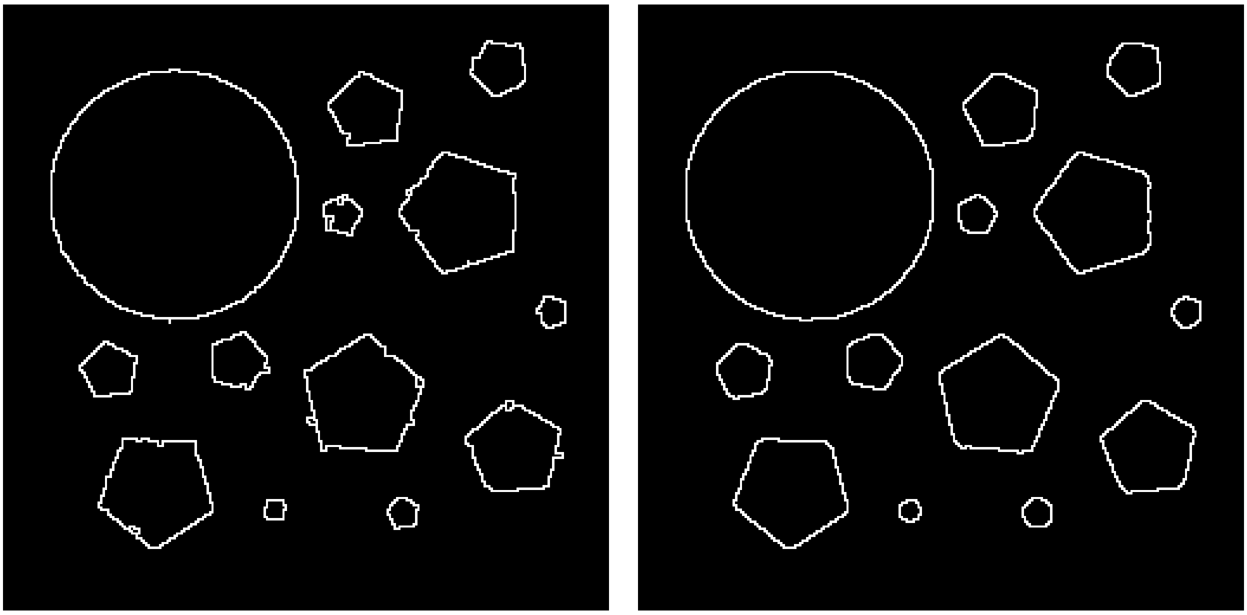


Figure of Merit
PSNR
MG 3x3
MG 5x5
SOBEL 3x3
SOBEL 5x5
MCD 5x5 with r=8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
5
10
15
20
25
30
35

Fig. 2



(a) (b)

Fig. 3